Circular Polarization from Gamma-Ray Burst Afterglows

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ABSTRACT

We investigate the circular polarization (CP) from Gamma-Ray Burst (GRB) afterglows. We show that a tangled magnetic field cannot generate CP without an ordered field because there is always an oppositely directed field, so that no handedness exists. This implies the observation of CP could be a useful probe of an ordered field, which carries valuable information on the GRB central engine. By solving the transfer equation of polarized radiation, we find that the CP reaches 1% at radio frequencies and 0.01% at optical for the forward shock, and 10-1% at radio and 0.1-0.01% at optical for the reverse shock.

Subject headings: gamma rays: bursts — gamma rays: theory — polarization — radiation mechanisms: non-thermal — shock waves

1. INTRODUCTION AND SUMMARY

Recently a very large linear polarization (LP), $\sim 80\pm20\%$, in the prompt γ-ray emission of GRB 021206 was discovered (Coburn & Boggs 2003). The degree of LP was at the theoretical maximum of the synchrotron emission, which implies an uniform ordered magnetic field over the visible region (but see also Eichler & Levinson 2003 for the scattering origin of LP). Since the causally connected region is smaller than the visible one (Gruzinov & Waxman 1999), an ordered field could be advected from the central engine of the Gamma-Ray Burst (GRB) and even drive the GRB explosion (Coburn & Boggs 2003; Lyutikov, Pariev & Blandford 2003).

On the other hand, LP of $\lesssim 10\%$ (typically a few %) has been detected in the GRB afterglows (Covino et al. 2003 and references there in), which is attributed to synchrotron emission behind a shock (e.g., Mészáros 2002). In most popular models (Gruzinov 1999;

Ghisellini & Lazzati 1999; Sari 1999; Rossi et al. 2002), the magnetic field is generated at the shock front and completely tangled (Medvedev & Loeb 1999). LP arises due to the geometric asymmetry provided by the afterglow jet observed off-axis if the magnetic fields parallel and perpendicular to the jet have different strengths. The γ-ray LP mentioned above could be also explained in this model if the jet is very narrow (Waxman 2003; Nakar, Piran & Waxman 2003; but see also Granot 2003).

Thus the present issue is *whether an ordered magnetic field exists or not*. If an ordered field exists in afterglows, its fraction to a tangled field carries valuable information on the GRB central engine. In this Letter we show that observations of the circular polarization (CP) could be a useful probe of the ordered field. CP has been detected in AGN jets (Wardle et al. 1998; Homan & Wardle 1999; Bower et al. 2002) and X-ray binaries (Fender, et al. 2000, 2002) in recent years. Theoretically, these observations are explained by a plasma effect in synchrotron sources. We apply this theory to the GRBs for the first time.

There are two main mechanisms to generate CP: intrinsic CP of synchrotron emission and Faraday conversion (FC) in sources. FC is a plasma effect which converts LP into CP (e.g., Jones & O'Dell 1977a,b). These are treated all together by solving the transfer equation of polarized radiation in §2. Then, we show that *the tangled field cannot generate CP* in §3. Next we estimate CP from GRB afterglows in presence of an ordered field together with a tangled field in §4. We find that, if the ordered field is comparable to or more than the tangled one, the degree of CP is about 1% at radio frequencies and 0.01% at optical for the forward shock, and it reaches 10-1% at radio and 0.1-0.01% at optical for the (early) reverse shock. The radio CP of the reverse shock remains to be $\sim 1\%$ even if the ordered field is weak (1% of the tangled one). CP in reverse shock radio emission of GRB 990123 has an upper limit of 37% at the 99.9% confidence level (Kulkarni et al. 1999; see also Finkelstein, Ipatov & Gnedin 2002). Further observation of CP will be a diagnosis of the ordered field and bring clues to the nature of the GRBs.

2. TRANSFER EQUATION

We first consider the evolution of the Stokes parameters, I, Q, U, V, (in units of erg s^{-1} cm^{-2} sr^{-1} Hz^{-1}) in a homogeneous plasma with a weakly anisotropic dielectric tensor (Sazonov & Tsytovich 1968; Sazonov 1969a,b; Jones & O'Dell 1977a; Melrose 1980). It is

described by the transfer equation of the polarized radiation,

$$\begin{pmatrix} d/ds+\kappa_I & \kappa_Q\cos 2\phi & -\kappa_Q\sin 2\phi & \kappa_V \\ \kappa_Q\cos 2\phi & d/ds+\kappa_I & \kappa_V^* & \kappa_Q^*\sin 2\phi \\ -\kappa_Q\sin 2\phi & -\kappa_V^* & d/ds+\kappa_I & \kappa_Q^*\cos 2\phi \\ \kappa_V & -\kappa_Q^*\sin 2\phi & -\kappa_Q^*\cos 2\phi & d/ds+\kappa_I \end{pmatrix}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I \\ \eta_Q\cos 2\phi \\ -\eta_Q\sin 2\phi \\ \eta_V \end{pmatrix}, \quad (1)$$

where ϕ is the azimuthal projection angle of the magnetic field on the plane perpendicular to the line of sight (see Figure 1), $\kappa_{(I,Q,U,V)}$ are the absorptivity, $\eta_{(I,Q,U,V)}$ are the emissivity, and κ_Q^* and κ_V^* are rotativity and convertibility, respectively. These coefficients for a relativistic plasma with a power-law energy distribution have been derived by Sazonov (1969a,b) for a particular frequency region, $\nu_{min} \ll \nu$. We have extended the frequency region to $\nu \ll \nu_{min}$, and both cases are summarized in Appendix A. For the emissivity $\eta_{(I,Q,U,V)}$, we consider only the synchrotron emission.

There are mainly two ways to generate the circularity V from synchrotron sources. The first one is intrinsic emission due to the emissivity η_V (Legg & Westfold 1968). The second is FC (e.g., Jones & O'Dell 1977a), which is the conversion of Q and U into V by means of κ_Q^* in equation (1). If the natural modes of a plasma are nearly circular, the LP vector of propagating radiation rotates since the left- and right-circular modes have different phase velocities due to birefringence. This effect is well known as Faraday Rotation (FR) (Rybicki & Lightman 1979). FC is a similar phenomenon caused by birefringence of a medium. If the natural modes are linearly or elliptically polarized, the difference in the phase velocities leads to the cyclic conversion between CP and LP. FC becomes effective around the self-absorption frequency since $|\kappa_Q^*/\kappa_I| \sim 1$ for $p \sim 2$ from equations (A1) and (A3). Note that, if the magnetic field is uniform, the synchrotron emission does not generate U in the coordinate system with $\phi = 0$. Therefore FC does not occur without the rotation of Q into U by κ_V^*.

When we consider the tangled magnetic fields later, the coefficients in equation (1) are averaged with respect to the distribution of the magnetic field. This is justified when the typical scale over which the Stokes parameters change is much larger than that of the field orientation and hence the correlations between the Stokes parameters and the transfer coefficients tend to zero (Ruszkowski & Begelman 2002). [1]

[1] In this Letter we also neglect the coupling of the natural modes in an inhomogeneous medium (Jones & O'Dell 1977b; for its validity see Ruszkowski & Begelman 2002).

3. MAGNETIC FIELD CONFIGURATION

Since we know very little about the magnetic field configuration in the afterglows of the GRB's, we here consider two components, i.e., the ordered field and the axisymmetric tangled field. There is a preferred direction in which the fluid moves. Therefore, the assumption of the axisymmetry of the tangled field is quite natural. The ordered field is expected in the patchy model (Gruzinov & Waxman 1999), or if the magnetic field advected from the central engine prevails or an ordered magnetic field exists in the medium into which the shock propagates (Granot & Königl 2003). The tangled field could be produced by the Weibel instability (Medvedev & Loeb 1999) or the turbulence behind the shock.

We set a coordinate system in the shocked fluid frame as shown in Figure 1. The z-axis is the radial direction, in which the fluid moves. The ordered field is characterized by the strength B_{ord} and the direction $(\vartheta_{ord}, \varphi_{ord})$. The axisymmetric tangled field is described by the field strength as a function of ϑ_{tan}, $B_{tan}(\vartheta_{tan})$, and the probability per unit solid angle $f(\vartheta_{tan})$. According to Sari (1999), we adopt $B_{tan}(\vartheta_{tan}) \propto (\xi^2 \sin^2 \vartheta_{tan} + \cos^2 \vartheta_{tan})^{-1/2}$ and $f(\vartheta_{tan}) \propto B_{tan}^3(\vartheta_{tan})$. If $\xi \gg 1$, $\langle B_\parallel^2 \rangle \gg \langle B_\perp^2 \rangle$, and vice versa, where $B_\parallel$ and $B_\perp$ are the tangled magnetic field components parallel and perpendicular to the z-axis, respectively. We parametrize the ratio of the ordered field to the tangled one by $\zeta = B_{ord}^2/(\langle B_\parallel^2 \rangle + \langle B_\perp^2 \rangle)$. The total strength $B^2 = B_{ord}^2 + \langle B_\parallel^2 \rangle + \langle B_\perp^2 \rangle$ is determined by the afterglow model in the next section.

Given a magnetic field geometry and the plasma parameters from the afterglow model, we can take an angular average of the coefficients and solve the transfer equation (1) to obtain the Stokes parameters. Although the shock structure may be important (e.g., Ioka 2003), we neglect it as a first step. We define the observed frequency as the frequency in the fluid frame multiplied by the Lorentz factor of the fluid in the lab frame γ, whereas ratios of the Stokes parameters are Lorentz invariant.

If $\zeta = 0$ (no ordered field), we can immediately find that $\langle \eta_Q \sin 2\phi \rangle$, $\langle \eta_V \rangle$, $\langle \kappa_Q \sin 2\phi \rangle$, $\langle \kappa_Q^* \sin 2\phi \rangle$, $\langle \kappa_V \rangle$ and $\langle \kappa_V^* \rangle$ vanish as a result of the axisymmetry and reflection symmetry about the xy plane. *Therefore the tangled field alone cannot generate CP*. Intuitively this is because there is always an oppositely directed pair of magnetic fields, so that no handedness exists.

4. APPLICATIONS TO GRB AFTERGLOWS

First we consider the forward external shock with energy E propagating into a constant surrounding density n. According to the standard afterglow model (Sari, Piran & Narayan

1998), the Lorentz factor of the shocked fluid and the radius of the shock evolve as $\gamma = (17E/1024\pi n m_p c^5 t^3)^{1/8}$ and $R = (17Et/4\pi m_p nc)^{1/4}$, respectively, where t is the observer time. The shell thickness in the shocked fluid frame can be estimated by R/γ, which we use as the path length of the transfer equation (1). We assume that electrons are accelerated in the shock to a power law distribution of Lorentz factor γ_e, $N(\gamma_e)d\gamma_e \propto \gamma_e^{-p} d\gamma_e$ for $\gamma_e > \gamma_{min}$, where $N_e = \int_{\gamma_{min}} N(\gamma_e)d\gamma_e = 4\gamma n$ is the electron number density in the shocked fluid frame and $p > 2$. The minimum electron energy and the magnetic field strength in the shocked fluid frame are given by $\gamma_{min} = \epsilon_e [(p-2)/(p-1)] (m_p/m_e)\gamma$ and $B = (32\pi m_p \epsilon_B n)^{1/2}\gamma c$, respectively, where the parameters ϵ_e and ϵ_B are fractions of shock energy that go into the electrons and the magnetic energy, respectively. Thus we can calculate γ, γ_{min}, B, R/γ and N_e as a function of t for given E, n, p, ϵ_e and ϵ_B. We adopt $E = 10^{52}$ erg, $n = 1$ proton cm^{-3}, $p = 2.2$, $\epsilon_e = 0.1$ and $\epsilon_B = 0.01$ (Panaitescu & Kumar 2002). For simplicity, we temporarily neglect the jet effects.

The typical synchrotron frequency is $\nu_m = eB\gamma_{min}^2\gamma/2\pi m_e c = 145\epsilon_{e,-1}^2 \epsilon_{B,-2}^{1/2} E_{52}^{1/2} t_{1\mathrm{day}}^{-3/2}$ GHz where the convention $Q = 10^x Q_x$ is used except for $t_{1\mathrm{day}} = t/1$ day. For $\nu > \nu_m$ and $\nu < \nu_m$, we use equations (A1) and (A3), respectively. By using equation (A3), we can estimate the self-absorption frequency from $\kappa_I R/\gamma = 1$ as $\nu_a = 41\epsilon_{e,-1}^{-1}\epsilon_{B,-2}^{1/5} E_{52}^{1/5} n^{3/5}$ GHz, where we put $\sin\theta = 1$. The flux is given by $F_\nu \propto t^{1/2}\nu^\beta$ with $\beta = 2$ and $1/3$ for $\nu < \nu_a$ and $\nu_a < \nu < \nu_m$, respectively (Sari, Piran & Narayan 1998). We can neglect the electron cooling for our interests.

Figure 2 illustrates CP from a forward shock. The degree of CP is about 1% at the self-absorption frequency ν_a and 0.01% at optical for $\zeta \gtrsim 1$. We have found that the dependence of CP on t and ξ is weak, and the dependence on ζ, ϑ_{ord}, φ_{ord} and χ is roughly $\propto \sqrt{\zeta/(1+\zeta)}\cos\theta_{ord}$ (see Figure 1 for the relation between θ_{ord}, ϑ_{ord}, φ_{ord} and χ). The CP is mainly due to intrinsic emission.

Next we consider the reverse shock propagating the ejected shell itself (Mészáros & Rees 1997; Sari & Piran 1999a). Let γ_0 and T be an initial Lorentz factor of the shell and the burst duration, respectively. Then the Lorentz factor at the shock crossing time is given by $\gamma_\times = \min(\gamma_0, \gamma_c)$, where $\gamma_c \equiv (3E/32\pi n m_p c^5 T^3)^{1/8}$ is a critical Lorentz factor (Kobayashi 2000; Kobayashi & Zhang 2003; Zhang, Kobayashi & Mészáros 2003; Sari & Piran 1999a,b). The shock crossing time is given by $t_\times = (\gamma_0/\gamma_\times)^{8/3}T$. At $t = t_\times$ the minimum electron energy, the magnetic field strength and the electron number density in the shocked fluid frame are given by $\gamma_{min} = \epsilon_e [(p-2)/(p-1)] (m_p/m_e)\gamma_0/\gamma_\times$, $B = (32\pi m_p \epsilon_B n)^{1/2}\gamma_\times c$ and $N_e = 4\gamma_\times^3 n/\gamma_0$, respectively. After the shock crossing, these quantities approximately evolve as $\gamma \propto t^{-7/16}$, $\gamma_{min} \propto t^{-13/48}$, $B \propto t^{-13/24}$ and $N_e \propto t^{-13/16}$, respectively (Kobayashi 2000; Kobayashi & Sari 2000). The shell thickness in the shocked fluid frame is R/γ where $R =$

$(17Et/4\pi m_p nc)^{1/4}$. We adopt $\gamma_0 = 100$ and $T = 100$ s.

The typical synchrotron frequency is $\nu_m = 0.035\ \epsilon_{e,-1}^2\ \epsilon_{B,-2}^{1/2} n^{1/2}\ t_{1day}^{-73/48}\ T_2^{73/48}\ \gamma_{0,2}^2$ $\max[1, (\gamma_0/\gamma_c)^{73/18}]$ GHz. Using equation (A1) we can estimate the self-absorption frequency from $\kappa_I R/\gamma = 1$ as $\nu_a = 20\epsilon_{e,-1}^{2(p-1)/(p+4)}\ \epsilon_{B,-2}^{(p+2)/2(p+4)}\ n^{(p+5)/2(p+4)}\ t_{1day}^{-(73p+122)/48(p+4)}$ $T_2^{(73p+146)/48(p+4)}\ \gamma_{0,2}^2\ E_{52}^{1/2(p+4)}\ \max[1, (\gamma_0/\gamma_c)^{(73p-70)/18(p+4)}]$ GHz, where we put $\sin\theta = 1$. The flux peaks at $\nu \sim \nu_a$. We can neglect the electron cooling for our interests.

Figure 3 shows CP from a reverse shock. For $\zeta \gtrsim 1$, the degree of CP reaches 10-1% at radio frequencies, and 0.1-0.01% at optical. Even if $\zeta \sim 10^{-4}$, CP remains to be $\sim 1\%$ at the self-absorption frequency ν_a. This is because of the FC. The intrinsic CP decreases as ζ decreases, but V is generated from Q and U due to the FC. The dependence of CP on ξ is weak. If FC is not effective, the dependence on ζ, ϑ_{ord}, φ_{ord} and χ is roughly $\propto \sqrt{\zeta/(1+\zeta)}\cos\theta_{ord}$, while if FC is effective, it is not so simple.

So far we have calculated CP at one point on the afterglow image on the sky. The integration over the entire emitting region may suppress the observed CP as in the case of LP (Sari 1999; Ghisellini & Lazzati 1999). Let us estimate the suppression factor when we observe a jet with an opening angle Θ_0 from a viewing angle Θ_v. We consider the ordered magnetic field defined by $(\vartheta_{ord}, \varphi_{ord})$ in the coordinate with x-axis being the direction from the jet center to the line of sight on the sky and with z-axis being the direction in which the ejecta moves (see Figure 1). We assume that the viewable region is a uniform disk of angular extent $1/\gamma$ centered around the line of sight to the observer, since the afterglow image is rather homogeneous for $\nu \lesssim \nu_m$ (Granot, Piran & Sari 1999a,b) or after the jet break (Ioka & Nakamura 2001). Then, if we assume $V/I \propto \cos\theta_{ord}$ and take the coordinate origin at the line of sight on the sky, the suppression factor is given by $\int d\Theta d\Phi\,[\cos\vartheta_{ord}\cos\chi_\Theta + \sin\chi_\Theta \sin\vartheta_{ord}\cos(\varphi_{ord} - \Phi)]\,/\int d\Theta d\Phi$, where $\sin\chi_\Theta = 2\gamma\Theta/(1 + \gamma^2\Theta^2)$ and $\Theta_v^2 + 2\Theta_v\Theta\cos\Phi + \Theta^2 < \Theta_0^2$. Interestingly, no cancellation takes place when $\vartheta_{ord} = 0$ or π. Even when the cancellation occurs, e.g, in the case of $\vartheta_{ord} = \pi/2$, some amount of CP remains if the visible region has an asymmetry due to the jet geometry as in the case of LP.

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A. Transfer coefficients

We summarize the transfer coefficients in equation (1) at an angle θ to the magnetic field B (Sazonov 1969a,b; Sazonov & Tsytovich 1968; Melrose 1980). In this section we measure all quantities in the shocked fluid frame. We assume that the electron number density in the interval of the Lorentz factor $d\gamma_e$ is power-law $dN_e = N(\gamma_e)d\gamma_e = \tilde{N}_e\gamma_e^{-p}d\gamma_e$ for $\gamma_{min} \leq \gamma_e$ with $p > 2$. Then, at frequencies $\nu_{min} \equiv \gamma_{min}^2\nu_\perp \ll \nu$ the coefficients are given by

$$\begin{aligned}
\eta_I &= \eta_\alpha\eta_\perp(\nu/\nu_\perp)^{-\alpha}, \quad \eta_Q = \eta_\alpha^Q\eta_\perp(\nu/\nu_\perp)^{-\alpha}, \quad \eta_V = -\eta_\alpha^V\eta_\perp(\nu/\nu_\perp)^{-\alpha-1/2}\cot\theta, \\
\kappa_I &= \kappa_\alpha\kappa_\perp(\nu/\nu_\perp)^{-\alpha-5/2}, \quad \kappa_Q = \kappa_\alpha^Q\kappa_\perp(\nu/\nu_\perp)^{-\alpha-5/2}, \quad \kappa_V = \kappa_\alpha^V\kappa_\perp(\nu/\nu_\perp)^{-\alpha-3}\cot\theta, \\
\kappa_Q^* &= -\kappa_\alpha^{*Q}\kappa_\perp(\nu/\nu_\perp)^{-3}\gamma_{min}^{-2\alpha+1}\{1-(\nu/\nu_{min})^{-\alpha+1/2}\}, \\
\kappa_V^* &= \kappa_\alpha^{*V}\kappa_\perp(\nu/\nu_\perp)^{-2}(\ln\gamma_{min})\gamma_{min}^{-2(\alpha+1)}\cot\theta,
\end{aligned} \tag{A1}$$

where $\nu_\perp \equiv |e|B\sin\theta/2\pi m_e c$, $\eta_\perp \equiv (e^2/c)\tilde{N}_e\nu_\perp$, $\kappa_\perp \equiv (e^2/m_e c)\tilde{N}_e/\nu_\perp$, $\alpha \equiv (p-1)/2$, and

$$\begin{aligned}
\eta_\alpha &= \frac{3^{\alpha+1/2}}{4(\alpha+1)}\,\Gamma\left(\frac{\alpha}{2}+\frac{11}{6}\right)\Gamma\left(\frac{\alpha}{2}+\frac{1}{6}\right), \quad \eta_\alpha^Q = \frac{3^{\alpha+1/2}}{4(\alpha+5/3)}\,\Gamma\left(\frac{\alpha}{2}+\frac{11}{6}\right)\Gamma\left(\frac{\alpha}{2}+\frac{1}{6}\right), \\
\eta_\alpha^V &= \frac{3^\alpha(\alpha+3/2)}{2(\alpha+1/2)}\,\Gamma\left(\frac{\alpha}{2}+\frac{11}{12}\right)\Gamma\left(\frac{\alpha}{2}+\frac{7}{12}\right), \quad \kappa_\alpha = \frac{3^{\alpha+1}}{4}\,\Gamma\left(\frac{\alpha}{2}+\frac{25}{12}\right)\Gamma\left(\frac{\alpha}{2}+\frac{5}{12}\right), \\
\kappa_\alpha^Q &= \frac{3^{\alpha+1}}{4}\frac{\alpha+3/2}{\alpha+13/6}\,\Gamma\left(\frac{\alpha}{2}+\frac{25}{12}\right)\Gamma\left(\frac{\alpha}{2}+\frac{5}{12}\right), \\
\kappa_\alpha^V &= \frac{3^{\alpha+1/2}}{2}\frac{\alpha+2}{\alpha+1}\left(\alpha+\frac{3}{2}\right)\,\Gamma\left(\frac{\alpha}{2}+\frac{7}{6}\right)\Gamma\left(\frac{\alpha}{2}+\frac{5}{6}\right), \\
\kappa_\alpha^{*Q} &= \frac{\alpha+3/2}{4(\alpha-1/2)}, \quad \kappa_\alpha^{*V} = 2\,\frac{\alpha+3/2}{\alpha+1}.
\end{aligned} \tag{A2}$$

The above notations are the same as Jones & O'Dell (1977a; But κ_α^{*Q} is different. The appropriate integration of equation (9) in Sazonov (1969a) gives the above value). For the frequency region, $\nu \ll \nu_{min}$, the analogous representations have not been derived yet. Such a frequency region becomes important in the application to the forward shock of GRB afterglow. In this case, we obtain the following expressions:

$$\begin{aligned}
\eta_I &= \eta_\alpha\eta_\perp(\nu/\nu_\perp)^{1/3}\gamma_{min}^{-2\alpha-2/3}, \quad \eta_Q = \frac{1}{2}\eta_I, \quad \eta_V = -\eta_\alpha^V\eta_\perp\gamma_{min}^{-2\alpha-1}\cot\theta, \\
\kappa_I &= \kappa_\alpha\kappa_\perp(\nu/\nu_\perp)^{-5/3}\gamma_{min}^{-2\alpha-5/3}, \quad \kappa_Q = \frac{1}{2}\kappa_I, \quad \kappa_V = \kappa_\alpha^V\kappa_\perp(\nu/\nu_\perp)^{-2}\gamma_{min}^{-2(\alpha+1)}\cot\theta, \\
\kappa_Q^* &= \kappa_\alpha^{*Q}\kappa_\perp(\nu/\nu_\perp)^{-5/3}\gamma_{min}^{-2\alpha-5/3}, \quad \kappa_V^* = \kappa_\alpha^{*V}\kappa_\perp(\nu/\nu_\perp)^{-2}(\ln\gamma_{min})\gamma_{min}^{-2(\alpha+1)}\cot\theta,
\end{aligned} \tag{A3}$$

where

$$\eta_\alpha = \frac{3^{1/6}}{2(\alpha+1/3)}\,\Gamma\left(\frac{2}{3}\right), \quad \eta_\alpha^V = \frac{\pi}{3(\alpha+1/2)}, \quad \kappa_\alpha = \frac{3^{1/6}}{2}\frac{\alpha+3/2}{\alpha+5/6}\,\Gamma\left(\frac{2}{3}\right),$$

$$\kappa_\alpha^V = \frac{\pi(\alpha+3/2)}{3(\alpha+1)}, \quad \kappa_\alpha^{*Q} = \frac{3^{-1/3}}{4}\frac{\alpha+3/2}{\alpha+5/6}\,\Gamma\left(\frac{2}{3}\right), \quad \kappa_\alpha^{*V} = 2\,\frac{\alpha+3/2}{\alpha+1}. \tag{A4}$$

Note that the intrinsic LP (η_Q/η_I) in this frequency region is not $(p+1)/(p+7/3) \approx 70\%$ but 50%. In GRB 021206, considerable photons are below the break energy, i.e., $\nu \ll \nu_{min}$ (Coburn & Boggs 2003), so that even an ordered field might not explain the observed LP $\approx 80 \pm 20\%$.

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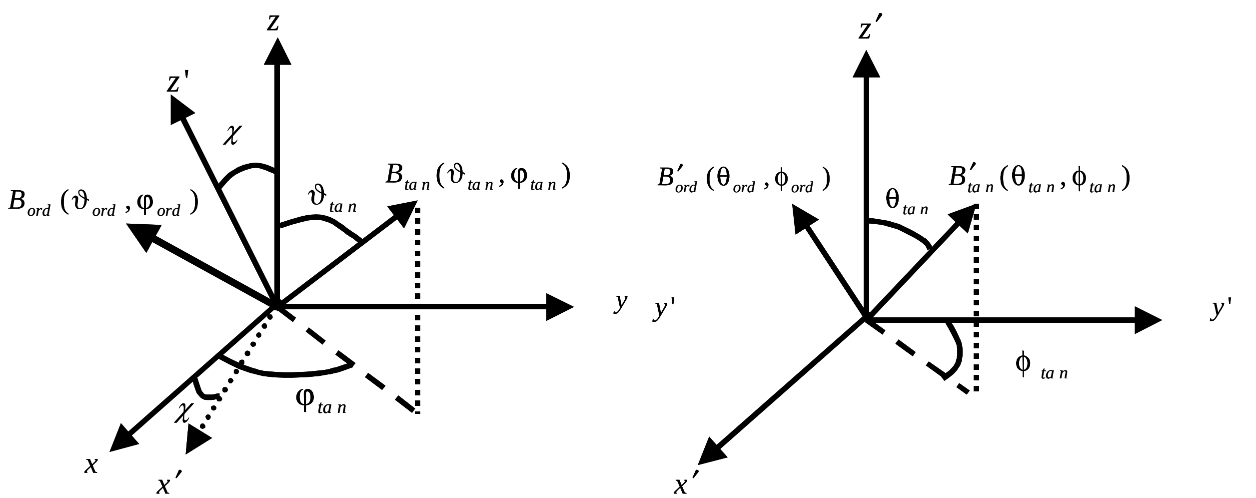

Fig. 1.— Magnetic field orientation in the shocked fluid frame. (*left panel*) The fluid flow is along the z axis. We introduce the ordered and tangled magnetic field. The former is defined by $(\vartheta_{ord}, \varphi_{ord})$, while the latter is stochastic and distributes as a function of (ϑ_{tan}, φ_{tan}). The observer is in the direction of the axis z', which is specified by the angle χ on the xz plane. (*right panel*) It is convienient to take a new coordinate in order to deal with the radiation transfer. In this coordinate, the magnetic field direction is specified by (θ, ϕ).

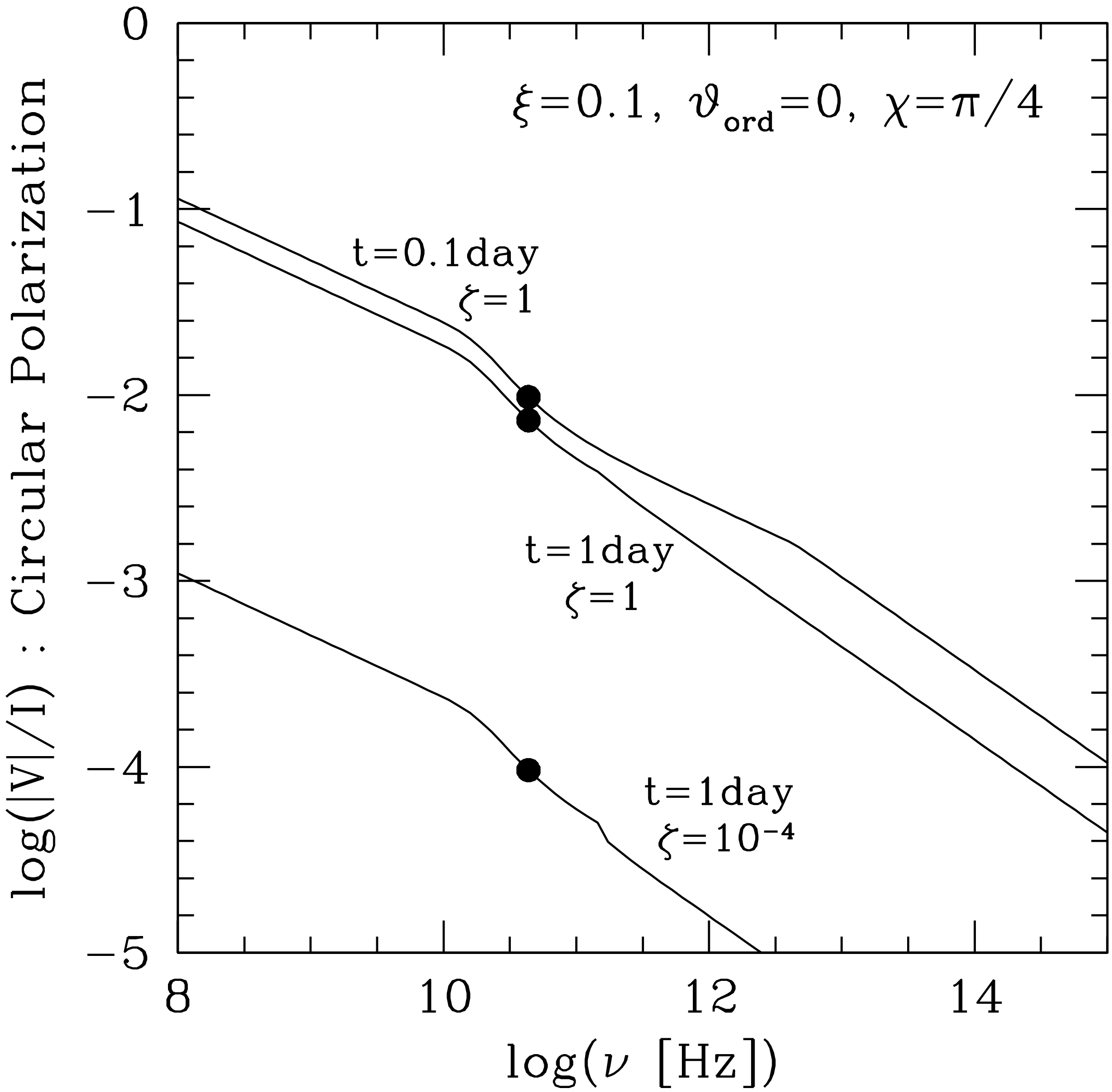

Fig. 2.— Circular polarization ($= |V|/I$) from a forward shock is shown as a function of the observed frequency ν for $t = 1$ and 0.1 day. The self-absorption frequencies ν_a are marked by black points. The ratio of the ordered field to the tangled one is defined by $\zeta = B_{ord}^2/(\langle B_\parallel^2\rangle + \langle B_\perp^2\rangle)$. The ordered field is directed to $\vartheta_{ord} = 0$. The perpendicular tangled component dominates the parallel one, i.e., $\xi = 0.1$. The observer is in the direction $\chi = \pi/4$ (see Figure 1). For this configuration, $V < 0$. We adopt $E = 10^{52}$ erg, $n = 1$ proton cm^{-3}, $p = 2.2$, $\epsilon_e = 0.1$ and $\epsilon_B = 0.01$.

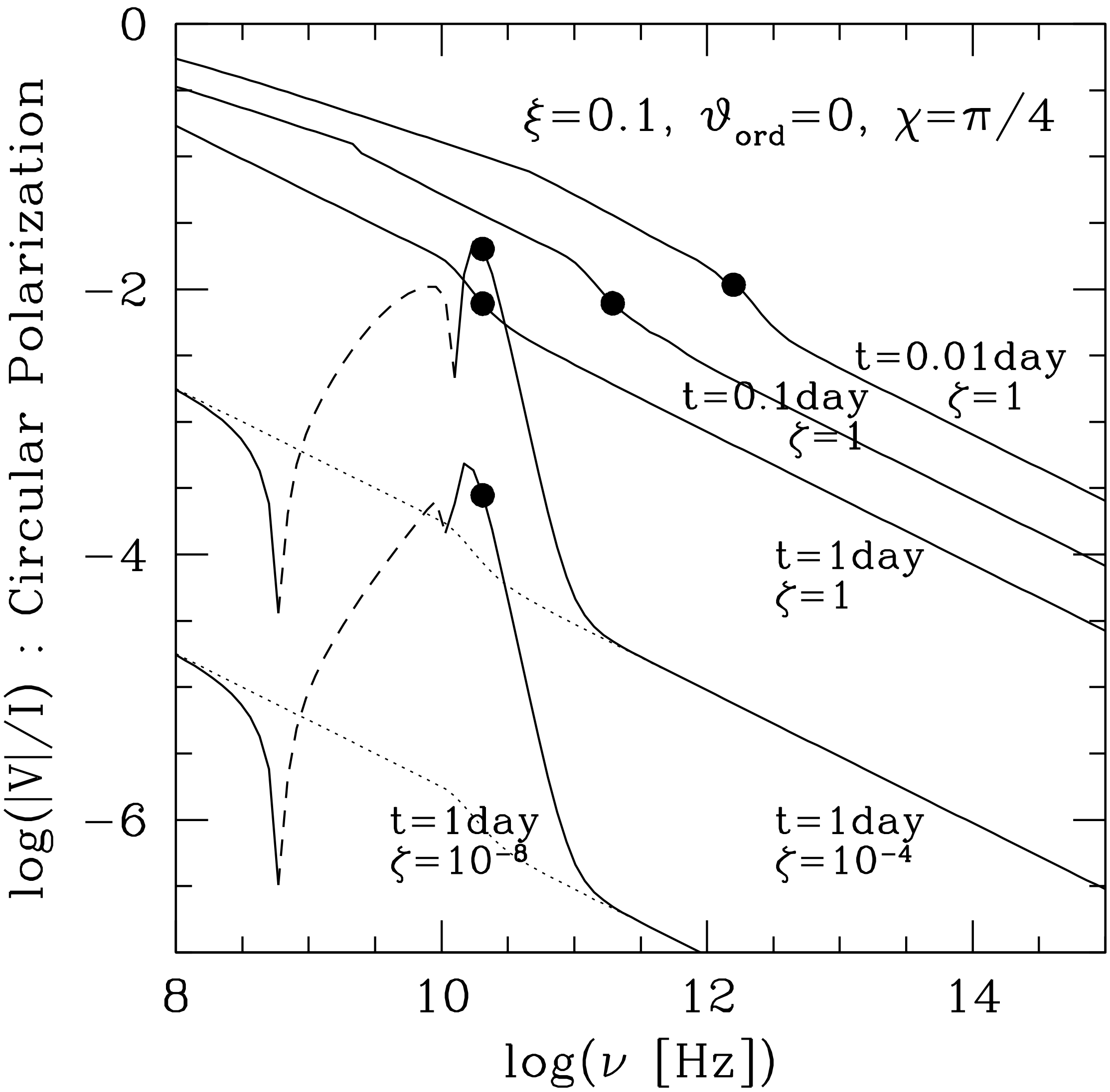

Fig. 3.— Circular polarization ($= |V|/I$) from a reverse shock is shown as a function of the observed frequency ν for $t = 1$, 0.1 and 0.01 day. For $V < 0$ ($V > 0$) we use solid (dashed) lines. The self-absorption frequencies ν_a are marked by black points. The dotted lines are calculated by putting $\kappa_Q^* = 0$ (no conversion). The ratio of the ordered field to the tangled one is defined by $\zeta = B_{ord}^2/(\langle B_\parallel^2\rangle + \langle B_\perp^2\rangle)$. We adopt $E = 10^{52}$ erg, $n = 1$ proton cm^{-3}, $p = 2.2$, $\epsilon_e = 0.1$, $\epsilon_B = 0.01$, $\gamma_0 = 100$, $T = 100$ s, $\vartheta_{ord} = 0$, $\xi = 0.1$ and $\chi = \pi/4$.